

6. (Team) Let X, Y be two real valued random variables such that $X - Y$ and X are independent, and that $X - Y$ and Y are independent. Show that $X - Y$ is almost surely constant.

7. (Team) For each n , let X_n be an exponential random variable with parameter $q_n > 0$:

$$\mathbb{P}[0 \leq X_n \leq t] = 1 - \exp(-tq_n).$$

Suppose that X_1, X_2, \dots are independent.

- (a) What is $\mathbb{E}[\exp(-X_n)]$?
- (b) Suppose $\sum 1/q_n < \infty$. Show that $\sum X_n < \infty$ almost surely.
- (c) Suppose $\sum 1/q_n = \infty$. Show that $\sum X_n = \infty$ almost surely.

8. (Team) Consider the numbers $1, 2, \dots, 12$ written around a ring as they usually are on a clock. A random walker starts at 12 and at each step moves at random to one of its two nearest neighbors (with probability half-half)

What is the probability that she will visit all the other numbers before her first returning back to 12.